EXTREME RETARDATION IN ARITHMETIC

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During the past five years spent in examining backward and problem children in the public schools, there have come to the writer's attention thirty-four cases of children who were extremely backward in the arithmetic of the curriculum. The teachers said of these children, "Has no number sense," "Can get no idea of arithmetic," "I can teach her anything but arithmetic." The cases discussed in this paper comprise all such children so reported, found to be normal in general mental ability and who were not having marked difficulty in any other subject of the curriculum of the grades of which they were members. It may be remarked parenthetically that no child has ever been reported as having special difficulty with arithmetic only who proved to be a mentally defective child.

The usual tests of mental ability, consisting largely of tests derived from the Healy-Fernald series and the Terman scale, were given these children. In addition, an examination of the child's ability with the subjects taught in his grade or previous grades was given to verify the teacher's report of his school standing. With the children who showed an ordinary understanding of the history and geography instruction and other subjects of the curriculum, tests of mental ability were reduced to a minimum. In all cases, however, the cross-line tests A and B of the Healy-Fernald series were given and with the children above ten, generally, the Code test. The latter tests are significant in testing the child's ability to retain and analyze visual imagery and concentrate attention upon representative material as opposed to the self-corrective concrete.

Immediate auditory memory was tested by the use of spoken digits and sentences. Persistent memory was tested through the

child's report of school subjects studied over a period of time, such as history, geography, stories read at some time in the past, etc.

The special examination in arithmetic consists of such problems as will test the child's knowledge of numbers and number relationships. The examination consists of six parts. In each part there are two phases of each test employed. The first is the mechanical element which may be learned as a feat of mechanical memory. The second is the element of understanding of the abstract mathematical idea involved.

The steps in this examination are made to coincide with the arithmetic curriculum of the public schools. This curriculum is so arranged that counting up to ten is taught in the first grade. In the second grade, the forty-five number facts are taught, with the signs, plus and minus, the multiplication tables of twos and threes, and the times sign. Simple one-step problems are a part of the work of this grade. In the third grade, the multiplication tables are taught with formal multiplication by one digit, and the processes of addition and subtraction with bridging of tens. In the fourth grade, the formal work consists of multiplication and division with multipliers and divisors of two digits. In the fifth grade, the formal work consists of fractions and fractional processes. In the sixth grade, decimal fractions are taught, together with their processes. In the seventh grade, the formal work consists of percentage and its applications to simple problems. In the eighth grade, the work consists of square root and the applications of the arithmetic learned in lower grades to practical problems of industrial and business life.

The children of this study represent various degrees of retardation. Assuming that six years was the age at which all the children entered school—in most cases it could not be definitely determined—seven were in the grades normal to their ages; nine were retarded one year; thirteen were retarded two years; five were retarded three years.

It was found in each of these cases that the child was not lacking in a concept of number. In each case the child understood the meaning of number. The least advanced of these children could count a row of objects to twenty or more, and hand any desired

number from a group before him; he could also make number combinations in some fashion by the use of counters. The more advanced individuals could perform some of the processes of the number work between the grades of two and eight. The problem then resolved itself into the question, "Why were these children so far behind in the arithmetic work of their grades?" The answer was found in many cases when an investigation was made of the child's health history in connection with his school history. The tabulations indicate the findings in these cases. In eight cases it was found that the special defect in arithmetic was correlated quite definitely with special periods of ill-health. The significant information concerning these cases is contained in Table I. The tables give the following items of information:

Beginning at the left, in the first column is found the initials of the child; in the second, sex; the third, the age in years and months; in the fourth, grade, "R" in parenthesis indicating that the grade is being repeated; in the fifth, those topics of the arithmetic work of the curriculum in which the individual's knowledge is markedly deficient or lacking entirely; in the sixth, those topics with which he has such a degree of knowledge as serves for a basis for the next step in the curriculum; in the seventh, the correlated circumstances indicating periods of ill-health and absence from school coinciding with the periods during which the topics indicated in the fifth or defect column are taught; in the eighth, information concerning the individual gained in later follow-up work.

Referring then to Table I, we find that C. B., a girl of eleven years and ten months, in Grade V, was lacking in knowledge of the processes of multiplication and division and in reasoning with arithmetical material; that she had, however, a knowledge of the multiplication tables, and also a knowledge of fractional ideas which was gained during the period of examination. The correlated circumstances in this case are long absence during the third grade with scarlet fever, and during the fourth grade with whooping cough. We can reasonably infer that the process of multiplication was therefore imperfectly or not at all acquired in the third grade, and likewise the process of long division was not mastered in the fourth grade. The recommendation was made

TABLE I

Special Periods of Ill-Health

Pupil	Sex	Age	Grade	Arithmetical Defects	Arithmetical Accomplishments	Correlated Circumstances	Follow-up
C. B.	F.	11–10	V	Multiplication; long division; reasoning	Tables; fractional ideas gained in one period of instruction	Scarlet fever in third grade; whooping cough in fourth	Seventh grade. A mediocre student, but passing
J. B.	M.	15	VIII B	Fractional processes; reduction of common fractions to decimals	Long division, slow but accurate; reasoning; fractional ideas	Out much during fifth with sore throat	Graduated with fifth-grade arithmetic
A. O.	F.	12	IV (R)	Long division; reasoning, though gets idea with teaching	Multiplication; subtraction	Pneumonia at six, nine, and ten years; influenza at eleven years. Open-air case	
D. B.	M.	11–7	V (R)	Long division; reasoning	Multiplication; subtraction; fractional processes	Out half of fourth grade; nervous; stomach trouble; poor vision	VI A. Good in long division and reasoning; poor in fractions; health improved; poor vision
W. P.	M.	10–9	IV	Pointing decimals in cost computing; bridging tens	Multiplication; reasoning	Influenza twice in third grade	
C. G.	M.	10–10	IV (R)	Combinations and processes not automatic; slow in work; trial and error method in long division	Reasoning; knowledge of processes; intelligent trial and error in all work	Pneumonia in third and fourth grades; poor nutrition; rachitic	
A. S.	M.	9–7	III (R)	Multiplication tables	Addition and subtraction combinations; mathematical conception of tables	Changed schools often; out much with toothache and earache	
P. P.	M.	9–11	IV B	Bridging tens, multiplication tables not automatic	Addition and subtraction combinations; process of long division	Scarlet fever in third grade	

that she be given special instruction in these topics. A follow-up of her case two years later showed her to be in the seventh grade, considered by her teacher a mediocre student in general, but passing in all subjects.

J. B. was defective in the process of common fractions and he could not reduce common fractions to decimals. He could do long division slowly but accurately. He could point off properly in division of decimals. He was able in reasoning with arithmetical material and possessed correct ideas of fractional quantities. He had been absent much during the fifth grade with sore throat, and therefore missed much of the instruction in common fractions given in that grade. It was recommended that he attend the fifth-grade arithmetic class. A year and one-half later he was reported as having graduated from the eighth grade, but having gotten no further than fifth-grade arithmetic.

For the other individuals of Table I, special periods of ill-health are shown to coincide with the grades in which were taught the particular topics of the child's failures.

Table II indicates a correlation between the type of work which the child hands in to the teacher and the child's general state of health. M. V. was a girl of twelve years and ten months, in the fifth grade. Her teacher said that she was very poor in arithmetic. The weekly papers, which were handed in and from which the arithmetic grades were made up, were always so poor and contained so many mistakes and problems not correctly done, that her grade was much below the passing mark. She did, however, upon examination do all the processes of arithmetic up to and including those of her grade, and was capable of reasoning with arithmetical material. She was suffering from tuberculosis, poor vision, much headache. She was not capable of prolonged application to work, and the arithmetic papers suffered because of this. Hers is a late case and has not been followed up.

A. P. was reported by his teacher as making grades of twenty or thirty usually. An examination of the papers handed in by him for a period of time indicated that he had always correctly performed the first two or three problems of the assignment, but with errors in the performance of the succeeding ones. It was apparent

TABLE II

GENERAL ILL-HEALTH

Pupil	Sex	Age	Grade	Arithmetical Defects	Arithmetical Accomplishments	Correlated Circumstances	Follow-up
M. V.	F.	12–10	V (R)	Poor grades	Normal; reasoning	Tuberculosis; poor vision; much headache	
M. B.	F.	9–3 12–4	II V B	Hands in absurd work	Normal; reasoning	Nervous; worries much; nutrition poor; history of chorea	Late case for arithmetic
F. D.	M.	10–9 12–8	IV V	Inaccurate in long division; fractional processes; tables not automatic	Process of long division; fractional ideas	Nervous trouble from early childhood; changed schools and grades often	Late case for arithmetic
S. M.	F.	14–2	VII (R)	Reasoning; decimals	Long division; common fractions	Much headache; tired much; poor nutrition	Technical High School
C. C.	M.	13–5	VI	Long division; fractional processes	Subtraction, multiplication; fractional ideas	Poor vision; headaches; open-air case	
W. M.	M.	14–6	VI	Poor grades; uncertain in reasoning and processes	Long division; fractions	Weak heart; poor nutrition; fourteen different schools	VII. Health improved. Application to work improved. Satisfactory
A. P.	M.	12–2	V	Poor grades; long division	Multiplication; fractions	Poor nutrition; inadequate food supply; listless; fatigues; first problems of written work correct, others wrong	VI A; VII A. Satisfactory work; health good; better food

upon observation of him at work that he fatigued very quickly, with consequent let down of effort. He could work at a task for no more than two or three minutes before fatigue was shown in restlessness, a desire to leave off effort, or sighing. An account of his food supply indicated that it consisted mostly of coffee and white bread. Conference with the mother resulted in a better dietary and his work was satisfactory in the next two grades.

Table III contains the data of six children who confessed a dislike of arithmetic or showed indolence or lack of interest in working with it. E. J. disliked very much to do the careful and intricate work necessary to produce accurate results. He worked outside of school hours in his father's grocery store, and had a fair ability to do sums in his head. He solved the problem, "If four dozen apples cost $1.50, what will three apples cost?" with the result, "9 and a fraction cents, it would be 10 cents." He could not be induced to put down all the work and finish it up properly. He could do simple problems in fractional processes without the use of paper, and with such problems as he could not do in this way he was only with difficulty induced to do so much of paper work as would yield a correct result. The industrial motive and the habits acquired therefrom were stronger than those of the school in setting his attitude toward the subject. It was only with reference to the former that arithmetic had acquired "meaning" for him. A follow-up of this boy in the eighth grade disclosed this characteristic still persisting. His teacher finds it difficult to induce him to hand in careful work.

Three cases are deserving of special mention since it is quite possible that they indicate a stage of difficulty that may be the lot of many other children in their progress in arithmetic. These three children evinced precisely the same type of trouble. Two were in the third grade and one in the second grade, but this second grade was doing the work normally done by the third grade in other schools. The difficulty which these three present is a lack of discriminative understanding of the meanings of the plus and times signs, and, therefore, a lack of understanding of the mathematical meaning of the multiplication tables. M. W. when first seen was eight years and eleven months of age. She was

TABLE III

LACK OF INTEREST, APPLICATION, ETC.

Pupil	Sex	Age	Grade	Arithmetical Defects	Arithmetical Accomplishments	Correlated Circumstances	Follow-up
C. B.	M.	14–8	VII A	Poor grades	Normal	Does not care to work; "Can do better but have not gotten down to business with arithmetic"	Finishing eighth in technical high school
N. D.	F.	12–5	V (R)	Long division; tables	Addition and subtraction combinations	Parochial school; little arithmetic	
T. K.	M.	14–1	VI A	Decimals; fractional processes	Long division; reasoning	Previously incorrigible	Returned to fifth for arithmetic; progress good
E. J.	M.	{ 7–11 11–8	II VI	Poor grades; inaccurate work	Normal	Indifferent, careless, does not like intricate work	VIII. Characteristic attitude as before
J. F.	M.	13–11	V (R)	Long division; fractions; tables not automatic	Mathematical conception of fractions and tables	Parochial school between ten and thirteen years; much interested in history and general reading	Working, accountant of bills of lading; likes it; left school in seventh grade; fourth grade arithmetic
P. W.	M.	10–7	IV A	Long division; multiplication	Addition and subtraction combinations; tables	Third grade residence ten weeks	
A. K.	M.	9–8	II (R)	Combinations not automatic	Mathematical conception	Shy, unresponsive; copies arithmetic work from neighbor	

G. H.	M.	14–4	VI B	Poor grades	Normal	Impulsive, flighty, does not like to concentrate; much general reading and information	
M. J.	F.	9–7 12–11	III B V(R)	Addition and subtraction combinations not automatic; does no work; inaccurate in long division	Mathematical conception; fractional ideas; reasoning; tables automatic	Lazy, likes to read light literature; afraid to try. "I doubt myself too much"	None since V A
W. S.	M.	12–7	V	Poor grades; inaccurate in all processes; confusion in bridging tens	Tables; fractional ideas	Left-handed with right-hand instruction	
M. C.	F.	8–7	III B	Absurd work in multiplication	Addition and subtraction combinations	Foreign language difficulty	
M. W.	F.	8–11	III(R)	Mathematical conception of multiplication tables; confusion of plus and times signs	Addition and subtraction combinations	Fond of reading, afraid of arithmetic; will not try	
K. I.	M.	9–6	IV B	Combinations and tables not automatic; confusion of plus and times signs	Mathematical conception		
G. F.	M.	8–11 9–7	II B II(R)	Combinations not automatic; confusion of plus and times signs	Mathematical conception	Does not like to concentrate on work; mother reads lessons and helps much	III A. Improved, normal progress

repeating Grade III B. Her teacher said, "She is absolutely lacking in number sense." On investigation, it was found that the child was intensely interested in dramatic types of literature and spent all of her time outside school hours in the public library reading fairy tales. The examiner gave a period of instruction in the mathematical meaning of the multiplication tables and of the times sign which she came to comprehend, though slowly. When she was inquired for later she had left the school and could not be found.

K. I., when seen, was nine years and six months of age, in Grade IV B. He had not made the number combinations of second-grade work automatic in memory and when given such problems, he found the answers by silent counting with nods of the head. He did not know the meaning of the times sign and confused it with the plus sign, and had become so distraught with regard to this that he was emotionally disturbed and unhappy when the subject of numbers was broached. When the "times" idea was explained by the use of marks he expressed much surprise. "It seems easy that way, I never thought of that." He said he had never seen any but the written symbol table and did not know what it meant.

The third case, G. F., presents the same type of difficulty. His teacher said, "He has no number sense." He was very fertile in his experimentation with number combinations given him. When given four plus three and four times three he multiplied, added and subtracted in an effort to find out what would satisfy the examiner's demand. He was taught the difference in the meaning of the signs and what they meant mathematically. He was then instructed to work out for himself the table of 2's. Two days later he had committed this table to memory. A recent report from the school indicates that he is doing satisfactory work.

W. S.[1] was found to be confused with reference to bridging tens. He sometimes did it properly and sometimes carried the units instead of the tens figure in multiplication. In the opinion of his examiner, this confusion was associated with the fact that the child was naturally left-handed, but that he had been taught to write

[1] Reports concerning this child and M. C. are contributed by Dr. Grace Munson, Department of Child Study, Chicago.

with the right hand. In such cases, it is apparent that children are frequently confused as to which hand to use under certain situations, or use them interchangeably in the same situations. This child's confusion was found to extend to certain manual situations also. All through school he had had difficulty in writing. His examiner reported in part:

> At present he shows very poor motor control for a boy of his age. He shows this in his drawing, in his writing of numbers, and his poor ability to draw designs from memory. I believe that the training of the right hand has confused his association in such a way that he lacks orientation in certain kinds of mental imagery. This defect shows up particularly in arithmetic processes in which it is necessary to imagine numbers in certain spatial relations to each other. He always carries the wrong number, though he is conscious of a tendency to do this and works valiantly to overcome it in the test problems given him. This lack of orientation makes his thinking confused and difficult.

One may assume this difficulty to be allied to the phenomenon of mirror-writing associated with left-handedness (1).

Many children who have failed to make the number knowledge of the second and third grades automatic resort to various methods of overcoming this defect in doing their work. D. G., of Table IV, in the third grade, who had made no number combinations automatic, resorted to long and elaborate systems of marks to solve her problems in division. She would first make a number of marks equal to the dividend, then fence off groups from this number equal to the divisor and so arrive at the quotient and the remainder. She had not learned the multiplication tables. Others who have not learned the multiplication tables do long division by a system of cumulative additions of the divisor instead of multiplying the divisor by the tentative quotient.

M. C., who did not know what multiplication meant, had worked out a system for doing the work of her grade which eminently satisfied herself. The work of the grade consisted in multiplying two-place numbers by one digit. She consistently added the multiplier and the number in the tens of the multiplicand and wrote this sum on the right-hand side below the line; then she doubled the number in the units place and placed the sum on the left-hand side of the first sum. She carried out this process

TABLE IV

REASON UNDISCOVERED

Pupil	Sex	Age	Grade	Arithmetical Defects	Arithmetical Accomplishments	Correlated Circumstances	Follow-up
S. R.	M.	14–3	VI	Inaccurate in long division; fractions; tables not automatic	Knowledge of process in long division; fractional ideas	Inmate of orphan asylum	VII B. Improved
D. G.	F.	9–10	III B	Combinations not automatic; carries out processes by use of marks	Arithmetical conception		III A. Satisfactory
S. K.	F.	14–9	VII A	Long division; fractions	Multiplication; knowledge of process of long division; fractional ideas		VIII B. Improved, will make grade
J. H.	M.	8–11	III A	Combinations not automatic	Arithmetical conception		
R. V.	F.	8–4	III B	Combinations not automatic	Arithmetical conception		

consistently in ten problems given to her. How she had come to originate this process and be so satisfied with it was not learned.

Dr. Bronner (2) devotes a chapter to the subject of special disability in arithmetic. She reported seven cases diagnosed as indicating such special disability. Case No. 7 was aged fifteen years and a half in the fourth grade. She reported that he failed in each of the four fundamental processes, could not add three-place numbers correctly, nor could he subtract, multiply, or divide. He had learned the multiplication tables by rote, but became confused in their use. He could not subtract 35 cents from 50 cents, nor 87 cents from $1.00. He could not make change with the money before him. He made change correctly only when handling nickels or multiples thereof. He could not solve the problem, "If one dozen costs 4 cents, what would two-thirds of a dozen cost?" He added change equaling $1.25 by counting by nickels and dimes. She reported that he had very poor auditory memory, but good visual memory. She said:

> Except in the light of his successes and failures on psychological tests, it would be difficult to explain his inability to learn arithmetic by ordinary methods. One could only have concluded without tests that he has a very specialized defect, but there could be no understanding of a basis for it. The explanation is evident when his exceedingly poor auditory memory for numbers is discovered. One may ask whether the boy had any concept of number. Certain it is he had opportunity for acquiring this if only through his experience while working. As for powers of discrimination and abstraction which are involved in the transition from experience with the concrete to facility with abstract number combinations, no defect for these is found on tests. If they are found at fault and are factors in his inability to perform arithmetic work, they are at least specialized and true only in this one field.

In her discussion of this case, Dr. Bronner indicates two debatable points. The first is, What shall we consider a concept of number? Second, Is it logical to attribute poor ability with numbers to poor auditory memory however authoritatively the latter may be established? With reference to the second point, one may say that observation of the teaching of number work in the schools indicates that it is taught through every avenue to the understanding. There is written number work placed on the blackboard by the teacher which the child copies and works out

on the paper before him. In the second grade he is given a textbook. He recites the number work orally and hears others around him reciting it. There are involved, therefore, visual imagery and speech and motor imagery as well as auditory imagery. In addition, there is involved the factor of drill which may reasonably be expected to exercise a corrective influence in its special field upon an innate inability in that field. The first point will be referred to again.

Dr. Bronner's case No. 9 was aged ten years and nine months in the second grade. Dr. Bronner says:

> Our examination of this little girl showed that she had no concept of number. She was utterly unable to master the work of her class because she had not the ability to grasp what was done. Whether she could have made normal progress had the concept of number been developed first through dealing with number relationships in concrete material we have no way of knowing, but we feel sure that without this the girl would become more and more confused by ordinary classroom procedure in this subject.

The child proved to be bright in all other reactions to tests of mental ability. The detailed report of the child's number work is as follows: Adds simple number combinations slowly; for instance, $6+5+9=20$; fails on subtraction and multiplication. Upon a second examination of this child eight months later, Dr. Bronner found that she could add four three-place numerals correctly. She could not subtract. When given a problem involving $7-7$, her answer was 7. Some examples of multiplication were correct and others failures; for instance, $4\times3=12$, $4\times8=32$, $4\times6=22$. She failed to give correct answers to 25 cents minus 8 cents, 25 cents minus 4, but gave the correct answer for 10 cents minus 6 cents. With actual change she could not add 50 cents and 25 cents, nor solve 50 cents minus 42 cents; given a quarter and asked to return the change after 18 cents are spent, she showed much difficulty in mental representation of the problem; counted the 18 cents in change and then tried to find what was needed to make up the 25 cents but, since it could not be done with the change before her, failed to solve the problem. Dr. Bronner does not state in what grade the child was at the time of this second examination. When she was in the second grade the year before

her number work was normal with reference to the second-grade curriculum in the Chicago public schools to which the child at this time presumably belonged. If at the time of the second examination she was in the third grade, much of the type of numbers given her for testing was in process of being taught to her, and she might reasonably not have been expected to be proficient in it all. The nonsensical response to the problem 7 minus 7 is one which is not infrequent. In such problems there is sometimes a type of misunderstanding, the child interpreting the problem to mean that he had 7 cents after having lost 7. This child, before coming to Dr. Bronner's attention, was an inmate of an orphan asylum and up to that time had had only two years of reliable schooling in the public school. The available school data in connection with the child's positive number ability would lead more justifiably to the conclusion of little educational opportunity rather than a special disability for arithmetic.

Case No. 11, after examination, was reported as very capable in all respects but very defective in number work. This child of eleven could not add, subtract, multiply, or divide; had only slight knowledge of multiplication tables, and could not carry out the process of multiplication in even very simple problems. When seen again, four years after the first examination, he was still very backward in arithmetic, according to the school report. With respect to his case, Dr. Bronner says, "We can only conclude that there is a special defect which makes it difficult for him to develop a concept of number."

Dr. Bronner in reporting these cases, did not define "concept of number." Such definition is necessary to this discussion. In order to arrive at a definition of this term, we may go back in the history of the development of the number idea in the mind of the child, to the point of its first beginnings. At what age and by what method does the child develop the idea of number?

There is much evidence to indicate that the normal child before entering school is possessed of some ideas of number. Hall (3) reports that out of 10,000 children entering school in Berlin, 7,265 knew numbers from 2 to 4. In Denmark, of 5,600 children entering school, 90 per cent of the boys and 80 per cent of the

girls knew numbers from 1 to 10. In Boston, of 300 children entering school, 17 per cent did not know numbers including 4; 8 per cent did not know numbers including 3.

Members of a class in genetic psychology reported data as follows: Of 183 five-year-old children two months in kindergarten where counting was not taught, 2 per cent could count less than 5, 12 per cent counted to 5, 14 per cent between 5 and 10, 7 per cent between 10 and 20, 61 per cent farther than 20. Of 45 four-year-old children under the same conditions, 13 per cent could count less than 5, 15 per cent could count up to and including 5, 30 per cent could count to between 5 and 10, 20 per cent between 10 and 20, and 17 per cent farther. Of 13 three-year-old children under the same circumstances, 2 could count less than 5, 3 to 5, 6 to between 5 and 10, 1 to 20, and one could not count at all. Fourteen two-and-a-half-year-old children under the same circumstances could count to some point less than 5. Terman (4) finds that children of four years of age can count to four.

It is evident, then, that most children have gained their first number experience some time before entering school. Recorded observations of the development of individual children indicate where this idea of number begins to develop and how it develops. Such data are reported by Perez, Scupin, Froebel, Major, Hogan, Dearborn, Stern, Pestalozzi, and other authors of child diaries. All such reports indicate the same type of process in gaining the idea of number.

The method of gaining the idea of number is most fully reported by Hogan and Major (6). Major's child, in the thirty-first month, knew 2 and 3 as numbers. Before that, he had been counting as a word series. He had been taught by his father and a playmate. In the thirty-second month he confused 3 and 4, and at the end of the third year was not certain about groups of 3 and 4 and was not interested in it.

The child reported by Hogan (7) in the twenty-first month counted 3, 4, 5, 6, 9 voluntarily. A nurse sometime before had counted cards for him, 1, 2, 3, 4. He at once picked up the cards 3 and 4 and after that had called his cards "fee fours." He had learned the number series so that when it was being counted for

him at 5 he would say "6," and "10" after he had heard 9. In the twenty-fifth month he knew 1, 2, and 3 as numbers; that is, he could select from a group these numbers of objects. In the sixty-first month he knew that 2 plus 2 is 4, counted up to 14 by 2's and had learned a great deal more about number ideas and relationships up to 100. He had worked out the multiplication table of 2's, had learned to count to 1,000, etc. In the beginning he had didactic teaching of the counting series from 1 to 10. The rest of his number knowledge was obtained through instruction which he himself sought. That is, after having discovered number meanings between 1 and 10, he asked for further number instruction as step by step was accomplished.

All the accounts of the beginnings of children's number ideas record that counting is first learned as a mere word series.

Pestalozzi, in his diary, reported that his two-and-one-half-year-old child knew number names but not their meanings. In characteristic fashion, Pestalozzi reproached himself for this:

> Why have I been so foolish as to let him pronounce important words without taking care at the same time to give him a clear idea of their meaning? Would it not have been more natural not to teach him to say three till he had thoroughly understood the meaning of two, and is it not in this way that children should be taught to count. Oh, how far I have erred from nature's paths in trying to improve on her teaching.

Observation of small children in the home, as well as diaries of child development indicate, however, that the method Pestalozzi so much deplored is the universal method. The child is first taught the number series by some older member of the family and he learns this as a mere non-sense series just as he learns eeny, meeny, miny, mo, etc. He is not, however, permitted to remain in this meaningless state of mind with reference to the number series. Members of the family frequently ask him, "How many?" always answering for him, so that he comes to associate the number terms with the term "how many," and to answer in some one of those terms. He learns, too, that he must produce some number of a group of objects before him when this demand is presented. He at first produces a number experimentally. If asked for two he produces a handful or three or what not. The instructor then

says, "No, that is not two; this much is two." This experience repeated over and over leads the child to comprehend "twoness," to use Dearborn's (8) term. At this point he has not a number concept, but a number percept. The same kind of experience must be repeated before he comprehends "threeness," and "fourness." Various reports indicate that this comprehension of the number meanings occurs about the age of four, varying between three and five. At the point where the child comprehends that the number series can be extended at will and applied universally to objects about him, he has gained a concept of number. The child who can count and who can report upon the number of objects he has counted and who can produce certain quantities upon demand from a larger group before him must be said to be possessed of the concept of number. The child who under favorable circumstances has failed to learn so much belongs to the imbecile group of feeble-mindedness.

The cases reported by Dr. Bronner, then, as lacking in concept of number, cannot strictly be so classified. A further investigation of these children with reference to their school experience might have indicated why they were backward in their number work, since the process is one of educational development. The number defects of the children reported by Dr. Bronner and the writer are the results of defects in the educational process rather than a defect in some part of the child's innate mental makeup.

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